**Scientific method**:

* Observe
* Hypothesize
* Predict
* Verify
* Validate
* Experiments must be *reproducible*
* Hypothesis must be *falsifiable*

**log-log plot**: uses logarithmic scales on both horizontal and vertical axes.

Knuth—the total running time of a program is determined by two primary factors:

* the cost of executing each statement
* the frequency of execution of each statement

**Tilde notation (~):**

We write ~f(N) to represent any function that, when divided by f(N), approaches 1 as N grows, and we write g(N) ~ f(N) to indicate that g(N)/f(N) approaches 1 as N grows.

Tilde approximation

Order of Growth:

|  |  |  |  |
| --- | --- | --- | --- |
| Constant | 1 | Statement | Add two numbers |
| Logarithmic | logN | Divided in half | Binary search |
| Linear | N | Loop | Find the max |
| Linearithmic | NlogN | Divided and conquer | Mergesort |
| Quadratic | N2 | Double loop | Check all pairs |
| Cubic | N3 | Triple loop | Check all triples |
| Exponential | 2N | Exhaustive search | Check all subsets |

* Develop a mathematical model:

1. input model
2. inner loop
3. cost model
4. analysis

* **Doubling ratio**: if T(n) ~aNb then T(2N)/T(N) ~2b
* *Caveats*: large constants, nondominant inner loop, instruction time (different memory location), system considerations, too close to call, strong dependence on inputs, multiple problem parameters.
* Worst-case performance guarantees vs. Amortized analysis (total cost of all operations divided by the number of operations)

Big-Oh [upper bound]

Let f and g be functions from the set of integers (or the set of real numbers) to the set of real numbers. Then f(x) is said to be ( g(x) ) , which is read as f(x) is big-oh of g(x) , iff there are constants C and n0 such that

| f(x) | C | g(x) |

whenever x > n0 .

Big-Omega [lower bound]

Let f and g be functions from the set of integers (or the set of real numbers) to the set of real numbers. Then f(x) is said to be ( g(x) ) , which is read as f(x) is big-omega of g(x) , if there are constants C and n0 such that

| f(x) | C | g(x) |

whenever x > n0 .

Big-Theta [optimal]

Let f and g be functions from the set of integers (or the set of real numbers) to the set of real numbers. Then f(x) is said to be ( g(x) ) , which is read as f(x) is big-theta of g(x) , if f(x) is O( g(x) ), and ( g(x) ) . We also say that f(x) is of order g(x) .

**Memory**

* Objects
* **Overhead**—includes a reference to the object’s class, garbage collection information and synchronization information (16 bytes)
* The memory usage is typically **padded** to be a multiple of 8 bytes.
* A nested non-static (inner) class such as our Node class requires an extra 8 bytes of overhead.

|  |  |
| --- | --- |
| Type | Bytes |
| int[] | ~4N |
| doube[] | ~8N |
| Date[] | ~40N |
| double[][] | ~8NM |